DETERMINATION OF ORBITS BY MEANS OF FOUR OBSERVATIONS

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According to Lagrange the expressions for the coordinates in the problem of two bodies can be put in the following form:

$$(1) \qquad u_i = f u_1 + g u'_1 \qquad (u = x,y,z)$$

Let us have four complete observations at the instants t_1, t_2, t_3 and t_4. By eliminating the velocities in the group (1): a) from the first and third equations; b) from the second and third equations, we shall have then two groups of equations:

$$(2) \quad \begin{aligned} g_4 x_2 &= g_2 x_4 + f' x_1 \\ g_4 y_2 &= g_2 y_4 + f' y_1 \\ g_4 z_2 &= g_2 z_4 + f' z_1 \\ f' &= f_2 g_4 - f_4 g_2 \end{aligned} \qquad (3) \quad \begin{aligned} g_4 x_3 &= g_3 x_4 + f'' x_1 \\ g_4 y_3 &= g_3 y_4 + f'' y_1 \\ g_4 z_3 &= g_3 z_4 + f'' z_1 \\ f'' &= f_3 g_4 - f_4 g_3 \end{aligned}$$

If we remember that:

$$x = \rho\lambda - X \qquad y = \rho\mu - Y \qquad z = \rho\nu - Z$$

where: $\lambda = \cos\alpha \cos\delta \quad \mu = \operatorname{sen}\alpha \cos\delta \quad \nu = \operatorname{sen}\delta$

we can eliminate in the group (2) the geocentric distance ρ_2 and in the group (3) the geocentric distance ρ_3 . We can write: ($k_i = \operatorname{cotg} \alpha_i$)

$$(4) \quad \begin{aligned} g_2(\lambda_4-k_2\mu_4)\rho_4 + f'(\lambda_1-\mu_1 k_2)\rho_1 &= g_4(k_2Y_2-X_2) + g_2(X_4-k_2Y_4) + f'(X_1-k_2Y_1) \\ g_3(\lambda_4-k_3\mu_4)\rho_4 + f''(\lambda_1-\mu_1 k_3)\rho_1 &= g_4(k_3Y_3-X_3) + g_3(X_4-k_3Y_4) + f''(X_1-k_3Y_1) \end{aligned}$$

From this system we obtain an equation in the unknown ρ_1. Adding to this equation the following: $r_1^2 = \rho_1^2 + R_1^2 + 2\rho_1 S_1$

we can solve the system:

$$\rho_1 = P + \frac{Q + g\rho_1}{r_1^3} \qquad r_1^2 = \rho_1^2 + R_1^2 + 2\rho_1 s_1$$

by means of an iterative process.

Next we calculate ρ_4 from (4) and $(x_2\ y_2\ z_2)$, $(x_3\ y_3\ z_3)$ from (2) and (3) respectively.

Finally we compute the velocities from the formulae:

$$\dot{u}_1 = \frac{f_2 u_4 - f_4 u_2}{f_2 g_4 - f_4 g_2}$$

REMARKS. It will be not convenient in general to take t_1 or t_4 as origin of the time, mainly owing to the lenght of the interval between the extreme observations. Besides in these cases such observations will be out of the interval used to calculate the observations.

However we used this scheme in our calculations for investigating their possible influences.

To the present subject Jekhowski and Roure have made some contributions.

THE ORBIT OF INO (173)

Observations: L.Boyer (Algiers).

U.T. 1954		α	δ
July	1.04559	303°06250	- 6°59686
July	21.96597	299°12737	- 8°62248
August	5.94212	296°04033	- 10°73358
August	25.88145	293°23254	- 13°76797

Sun Coordinates

	1	2	3	4
X	- 0.1522304	- 0.4857517	-0.6902529	-0.8927214
Y	0.9222733	0.8186990	0.6818577	0.4346262
	0.3999277	0.3550127	0.2956623	0.1884462

The comparison O - C

Coordinates and velocities of INO

$x_1 = 1.024475$	$\dot{x}_1 = 0.525629$
$y_1 = -2.262209$	$\dot{y}_1 = 0.378388$
$z_1 = -0.584830$	$\dot{z}_1 = 0.005824$

The series F and G

	F	G
2	0.996033	0.359405
3	0.988185	0.615037
4	0.971005	0.950975

O – C

Δα cos δ	–0°00008	–0°00006	–0°00006	–0°00009
Δδ	0°00002	–0°00033	–0°00010	–0°00002